

# Three-dimensional asymptotically flat Einstein-Maxwell theory

Glenn Barnich, Pierre-Henry Lambert and Pujian Mao

*Physique Théorique et Mathématique*

*Université Libre de Bruxelles and International Solvay Institutes*

*Campus Plaine C.P. 231, B-1050 Bruxelles, Belgium*

ABSTRACT. Three-dimensional Einstein-Maxwell theory with non trivial asymptotics at null infinity is solved. The symmetry algebra is a Virasoro-Kac-Moody type algebra that extends the bms3 algebra of the purely gravitational case. Solution space involves logarithms and provides a tractable example of a polyhomogeneous solution space. The associated surface charges are non-integrable and non-conserved due to the presence of electromagnetic news. As in the four-dimensional purely gravitational case, their algebra involves a field-dependent central charge.

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# 1 Introduction

The original studies of four-dimensional asymptotically flat spacetimes at null infinity [1–3] and their extensions to include the electromagnetic field [4, 5] rely on an expansion in inverse powers of the radial coordinate  $r$  for the metric components or the spin and tetrad coefficients. In order to guarantee a self-consistent solution space, some of these expansions need well-chosen gaps so as to prevent the appearance of logarithmic terms in  $r$ .

In more recent investigations, this assumption has been relaxed. More general consistent solution spaces have been proposed that involve double series with inverse powers and logarithms in  $r$  from the very beginning. Details on such “polyhomogeneous spacetimes” can be found for instance in [6–8].

Another non trivial aspect of 4d spacetimes with non trivial asymptotics at Scri is that charges associated to the asymptotic symmetry transformations, even though well-defined, are neither conserved nor integrable [9]. Furthermore, when considering a local version of the asymptotic symmetry algebra [10, 11], the associated current algebra acquires a field dependent central extension [12, 13].

In contrast, three-dimensional asymptotically flat Einstein gravity at null infinity is much easier, in the sense that the expansion in inverse powers of  $r$  of the general solution with non trivial asymptotics can be shown not to admit logarithms and to truncate after the leading order terms [11]. The symmetry algebra [14] is  $\mathfrak{bms}_3$ , the charges are conserved, integrable (and also  $r$  independent [15]), while their algebra involves a constant central extension [16], closely related to the one for asymptotically anti-de Sitter spacetimes [17].

The purpose of the present paper is to study three-dimensional Einstein-Maxwell theory with asymptotically flat boundary conditions at null infinity. This model allows one to illustrate several aspects of the four dimensional case in a simplified setting. On the one hand, there is a clear physical reason for the occurrence of logarithms as such a term is needed in the time component of the gauge potential in order to generate electric charge. This term leads to a self-consistent polyhomogeneous solution space that includes the charged analog of particle [18] and cosmological solutions [19–22]. The latter correspond to the flat space limit of the three-dimensional charged rotating asymptotically anti-de Sitter black holes [23]. On the other hand, the asymptotic symmetry algebra is a Virasoro-Kac-Moody type algebra that extends the  $\mathfrak{bms}_3$  algebra of the purely gravitational case. The associated surface charges turn out to be neither conserved nor integrable due to the presence of electromagnetic news. Furthermore the algebra of surface charges now involves a field dependent central charge that persists when switching off the news.

The plan of the paper is the following. In the next section, we work out the asymptotic symmetry algebra. In section 3, we present the polyhomogeneous solution space, while section 4 is devoted to the surface charges and their algebra. In the last section, we show that upon switching off the electromagnetic news, not surprisingly, charges become conserved and integrable. Nevertheless, both the asymptotic symmetry algebra and the central charge involve field dependent terms. The appendix contains details on intermediate computations that are omitted from the main text. Finally, a last section is devoted to a comparison of the novel results derived here in three and previous results obtained in four dimensions.

## 2 Asymptotic symmetries

To work out the asymptotic symmetries, we follow closely the original literature [24] and adapt it to the current context. More generally, for the Einstein-Yang-Mills system in all dimensions greater than 3, this problem has been addressed recently in detail in a unified way both for flat and anti-de Sitter backgrounds in [25]. In this approach, the gauge fixing condition in the definition of asymptotic flat spacetimes fix the radial dependence of gauge parameters completely, while the fall-off conditions fix the temporal dependence. In the current set-up, the fall-off conditions on  $A_u$  are more relaxed as compared to those considered in section 5.5 of [25] in order to account for non-vanishing electric charge. As a consequence, the time dependence of the electromagnetic gauge parameter is no longer fixed, unless one switches off the news.

In order to define asymptotic flatness of the three-dimensional Einstein-Maxwell at future null infinity, coordinates  $u, r, \phi$  are used together with the gauge fixing ansatz

$$g_{\mu\nu} = \begin{pmatrix} Ve^{2\beta} + r^2U^2 & -e^{2\beta} & -r^2U \\ -e^{2\beta} & 0 & 0 \\ -r^2U & 0 & r^2 \end{pmatrix}, \quad A_r = 0, \quad (2.1)$$

where  $U, \beta, V$  and  $A_u, A_\phi$  are functions of  $u, r, \phi$ . Suitable fall-off conditions that allow for non-vanishing electric charge are

$$\begin{aligned} U &= o(r^{-1}), \quad V = o(r), \quad \beta = o(r^0), \\ A_u &= O(\ln \frac{r}{r_0}), \quad A_\phi = O(\ln \frac{r}{r_0}), \end{aligned} \quad (2.2)$$

where  $r_0$  is a constant radial scale.

The gauge structure of Einstein-Maxwell theory can be described as follows. Gauge parameters are pairs  $(\xi^\mu, \epsilon)$  consisting of a vector field  $\xi^\mu \partial_\mu$  and a scalar  $\epsilon$ .

A generating set of gauge symmetries can be chosen as

$$-\delta_{(\xi, \epsilon)} g_{\mu\nu} = \mathcal{L}_\xi g_{\mu\nu}, \quad -\delta_{(\xi, \epsilon)} A_\mu = \mathcal{L}_\xi A_\mu + \partial_\mu \epsilon. \quad (2.3)$$

When the gauge parameters are field dependent, as will be the case for the parameters of asymptotic symmetries below, the commutator of gauge transformations contains additional terms:

$$[\delta_{(\xi_1, \epsilon_1)}, \delta_{(\xi_2, \epsilon_2)}](g_{\mu\nu}, A_\mu) = \delta_{[(\xi_1, \epsilon_1), (\xi_2, \epsilon_2)]_M}(g_{\mu\nu}, A_\mu), \quad (2.4)$$

where the Lie (algebroid) bracket for field dependent gauge parameters is defined through

$$\begin{aligned} [(\xi_1, \epsilon_1), (\xi_2, \epsilon_2)]_M &= (\hat{\xi}, \hat{\epsilon}), \\ \hat{\xi} &= [\xi_1, \xi_2] + \delta_{(\xi_1, \epsilon_1)} \xi_2 - \delta_{(\xi_2, \epsilon_2)} \xi_1, \quad \hat{\epsilon} = \xi_1(\epsilon_2) + \delta_{(\xi_1, \epsilon_1)} \epsilon_2 - (1 \leftrightarrow 2). \end{aligned} \quad (2.5)$$

Gauge transformations preserving asymptotically flat configurations are explicitly worked out in Appendix **A.1**. They are determined by gauge parameters depending linearly and homogeneously on arbitrary functions  $T(\phi), Y(\phi), E(u, \phi)$  according to

$$\begin{aligned} \xi^u &= f = T + uY', \\ \xi^\phi &= Y - f' \int_r^\infty \frac{e^{2\beta}}{r'^2} dr' = Y - \frac{f'}{r} + o(r^{-2}), \\ \xi^r &= -r\partial_\phi \xi^\phi + rUf' = -rY' + f'' + o(1), \\ \epsilon &= E(u, \phi) + f' \int_r^\infty \frac{e^{2\beta} A_\phi}{r'^2} dr' = E(u, \phi) + O\left(\frac{\ln \frac{r}{r_0}}{r}\right), \end{aligned} \quad (2.6)$$

where dot and prime denote  $u$  and  $\phi$  derivatives, respectively.

Consider then the “**bms**<sub>3</sub>/Maxwell” Lie algebra consisting of triples  $s = (T, Y, E)$  with bracket

$$[s_1, s_2] = (\hat{T}, \hat{Y}, \hat{E}), \quad (2.7)$$

where

$$\hat{T} = Y_1 T'_2 + T_1 Y'_2 - (1 \leftrightarrow 2), \quad \hat{Y} = Y_1 Y'_2 - (1 \leftrightarrow 2), \quad \hat{E} = Y_1 E'_2 + f_1 \dot{E}_2 - (1 \leftrightarrow 2). \quad (2.8)$$

This is the asymptotic symmetry algebra of the system in the following sense:

*When equipped with the modified bracket (2.5), the parameters (2.6) of the residual gauge symmetries form a representation of the Lie algebra (2.7).*

The proof, following the one originally worked out in [11], is sketched in Appendix **A.2**.

### 3 Solution space

In this section, we present the polyhomogeneous solution space for our model, following mainly [2, 7, 26].

We start from the Einstein-Maxwell Lagrangian density in three dimensions

$$\mathcal{L} = \frac{\sqrt{-g}}{16\pi G}(R - F^2), \quad (3.1)$$

with equations of motion

$$\partial_\nu(\sqrt{-g}F^{\mu\nu}) = 0, \quad L_{\mu\nu} := G_{\mu\nu} - T_{\mu\nu} = 0, \quad (3.2)$$

where  $T_{\mu\nu} = 2F_{\mu\rho}F_\nu{}^\rho - \frac{1}{2}g_{\mu\nu}F^2$ .

The detailed analysis in Appendix **A.3** then yields the following results: given the ansatz

$$A_\phi = \alpha(u, \phi) \ln \frac{r}{r_0} + A_\phi^0(u, \phi) + \sum_{m=1}^{\infty} \sum_{n=0}^m \frac{A_{mn}(u, \phi) (\ln \frac{r}{r_0})^n}{r^m}, \quad (3.3)$$

at a fixed time  $u_0$ , the general solution to the Einstein-Maxwell system in three dimensions with the prescribed asymptotics is completely determined in terms of the initial data  $A_\phi^0(u_0, \phi)$ ,  $A_{mn}(u_0, \phi)$ , the news functions  $A_u^0(u, \phi)$  and integration functions  $\omega(\phi)$ ,  $\lambda(\phi)$ ,  $\theta(\phi)$ ,  $\chi(\phi)$  according to

$$\alpha = -\omega - u\lambda', \quad N = \chi + u\theta' \quad (3.4)$$

$$\beta = -\frac{\alpha^2}{2r^2} + \sum_{m=1}^{\infty} \sum_{n=0}^m \frac{\beta_{mn} (\ln \frac{r}{r_0})^n}{r^{m+2}}, \quad (3.5)$$

$$U = \frac{4\lambda\alpha \ln \frac{r}{r_0} + 2\lambda\alpha - N}{2r^2} + \sum_{m=1}^{\infty} \sum_{n=0}^m \frac{U_{mn} (\ln \frac{r}{r_0})^n}{r^{m+2}}, \quad (3.6)$$

$$A_u = -\lambda \ln \frac{r}{r_0} + A_u^0 + \frac{\alpha'}{r} + \sum_{m=1}^{\infty} \sum_{n=0}^m \frac{B_{mn} (\ln \frac{r}{r_0})^n}{r^{m+1}}, \quad (3.7)$$

$$V = 2\lambda^2 \ln \frac{r}{r_0} + \theta + \frac{2\alpha\lambda' - 2\lambda\alpha'}{r} + \sum_{m=1}^{\infty} \sum_{n=0}^m \frac{V_{mn} (\ln \frac{r}{r_0})^n}{r^{m+1}}, \quad (3.8)$$

where the functions  $\beta_{mn}, U_{mn}, B_{mn}, V_{mn}$  are determined recursively in terms of the initial data, the news, the integration functions and their  $\phi$  derivatives. In particular,

$$\dot{A}_\phi^0 = -\lambda' + (A_u^0)'. \quad (3.9)$$

Furthermore, the leading parts of the metric and electromagnetic gauge potentials are given by

$$\begin{aligned} ds^2 = & [2\lambda^2 \ln \frac{r}{r_0} + \theta + O(r^{-1})] du^2 - [2 + O(r^{-2})] dudr \\ & - [4\lambda\alpha \ln \frac{r}{r_0} + 2\lambda\alpha - \chi - u\theta' + O(r^{-1} \ln \frac{r}{r_0})] dud\phi + r^2 d\phi^2, \end{aligned} \quad (3.10)$$

$$A_\phi = \alpha \ln \frac{r}{r_0} + A_\phi^0 + O(r^{-1} \ln \frac{r}{r_0}), \quad A_u = -\lambda \ln \frac{r}{r_0} + A_u^0 + O(r^{-1}). \quad (3.11)$$

Asymptotic symmetries transform solutions to solutions. This allows one to work out the transformation properties of the functions characterising asymptotic solution space:

$$\begin{aligned}
\mathcal{L}_\xi g_{uu} : \quad & -\delta\theta = Y\theta' + 2(\theta - \lambda^2)Y' - 2Y''', \\
& -\delta\lambda = Y\lambda' + \lambda Y', \\
\mathcal{L}_\xi g_{u\phi} : \quad & -\delta\chi = Y\chi' + 2(\chi - 2\omega\lambda)Y' + T\theta' + 2(\theta - \lambda^2)T' - 2T''', \\
& -\delta\omega = Y\omega' + \omega Y' + T\lambda' + \lambda T', \\
\mathcal{L}_\xi A_u + \dot{\epsilon} : \quad & -\delta A_u^0 = Y(A_u^0)' + (A_u^0 + \lambda)Y' + f\dot{A}_u^0 + \dot{E}, \\
\mathcal{L}_\xi A_\phi + \epsilon' : \quad & -\delta A_\phi^0 = Y(A_\phi^0)' + (A_\phi^0 - \alpha)Y' + f\dot{A}_\phi^0 + A_u^0 f' + E'.
\end{aligned} \tag{3.12}$$

## 4 Surface charge algebra

Associating charges to asymptotic symmetries in general relativity is a notoriously subtle question. The approach followed here consists in deriving conserved co-dimension 2 forms in the linearized theory that can be shown to be uniquely associated, up to standard ambiguities, to the exact symmetries of the background [27–29]. When using these expressions for asymptotic symmetries in the full theory, neither conservation nor integrability is guaranteed [9, 12, 13, 30].

More concretely, using the general expressions for the linearized Einstein-Maxwell system derived in [31], the surface charge one form of the linearized theory reduces to

$$\oint_{S^\infty} \delta k_{\xi, \epsilon} = -\delta \oint_{S^\infty} K_{\xi, \epsilon} + \oint_{S^\infty} K_{\delta\xi, \delta\epsilon} - \oint_{S^\infty} \xi \cdot \Theta, \tag{4.1}$$

where

$$\begin{aligned}
K_{\xi, \epsilon} &= (dx^{n-2})_{\mu\nu} \frac{\sqrt{-g}}{16\pi G} [\nabla^\mu \xi^\nu - \nabla^\nu \xi^\mu + 4F^{\mu\nu}(\xi^\sigma A_\sigma + \epsilon)], \\
\Theta &= (dx^{n-1})_\mu \frac{\sqrt{-g}}{16\pi G} [\nabla_\sigma \delta g^{\mu\sigma} - \nabla^\mu \delta g^\nu_\nu + 4F^{\sigma\mu} \delta A_\sigma],
\end{aligned} \tag{4.2}$$

and  $S^\infty$  is the circle at constant  $u = u_0$  and  $r = R \rightarrow \infty$ . At this stage, we have used already that expressions in  $\oint_{S^\infty} \delta k_{\xi, \epsilon}$  that are proportional to the exact generalized Killing equations vanish,

$$\begin{aligned}
\frac{1}{16\pi G} \oint_{S^\infty} \delta g^\nu_\rho (\nabla^\mu \xi^\rho + \nabla^\rho \xi^\mu) \sqrt{-g} (d^{n-2}x)_{\mu\nu} &= 0, \\
\frac{1}{4\pi G} \oint_{S^\infty} g^{\mu\rho} \delta A^\nu (\mathcal{L}_\xi A_\rho + \partial_\rho \epsilon) \sqrt{-g} (d^{n-2}x)_{\mu\nu} &= 0,
\end{aligned} \tag{4.3}$$

when evaluated for solutions and asymptotic symmetry parameters (2.6). As in four-dimensional asymptotically flat pure Einstein gravity [9], the remaining expression

then splits into an integrable part and a non-integrable part proportional to the electromagnetic news,

$$\oint_{S^\infty} \delta k_{\xi, \varepsilon} = \delta Q_s + \Theta_s, \quad (4.4)$$

with

$$\begin{aligned} Q_s[g, A] &= \frac{1}{16\pi G} \int_0^{2\pi} d\phi \left[ \theta T + Y(\chi + 4\lambda A_\phi^0) + 4\lambda E \right], \\ \Theta_s[\delta g, \delta A; g, A] &= \frac{1}{4\pi G} \int_0^{2\pi} d\phi f A_u^0 \delta \lambda. \end{aligned} \quad (4.5)$$

It follows that  $(8G)^{-1}\theta$ ,  $(8G)^{-1}(\chi + 4\lambda A_\phi^0)$ ,  $(2G)^{-1}\lambda$  can be interpreted as the mass, angular momentum and electric charge aspect, respectively.

Applying now the proposal of [12, 13] for the modified bracket of the integrable part of the charges,

$$\{Q_{s_1}, Q_{s_2}\} = -\delta_{s_2} Q_{s_1} + \Theta_{s_2}[-\delta_{s_1} g, -\delta_{s_1} A; g, A], \quad (4.6)$$

gives

$$\{Q_{s_1}, Q_{s_2}\} = Q_{[s_1, s_2]} + K_{s_1, s_2}, \quad (4.7)$$

where

$$K_{s_1, s_2} = \frac{1}{8\pi G} \int_0^{2\pi} d\phi \left[ Y_1' T_2'' - 2\lambda f_1 \dot{E}_2 - \lambda^2 T_1 Y_2' - (1 \leftrightarrow 2) \right]. \quad (4.8)$$

It is then straightforward to check that the field dependent central extension satisfies the generalised cocycle condition

$$K_{[s_1, s_2], s_3} - \delta_{s_3} K_{s_1, s_2} + \text{cyclic}(1, 2, 3) = 0. \quad (4.9)$$

## 5 Switching off the news

In the analysis above, we are in an unusual situation where the asymptotic symmetry algebra depends arbitrarily on time through the dependence of  $E$  on  $u$ . This can be fixed by requiring the electromagnetic news function to vanish,  $A_u^0 = 0$ . Since asymptotic symmetries need to preserve this condition, we find from (3.12) that  $E = \bar{E} - \int_{u_0}^u du' \lambda Y'$ . The asymptotic symmetry algebra then becomes time independent, but field dependent since the last of (2.8) gets replaced by

$$\hat{\bar{E}} = Y_1 \bar{E}_2' - T_1 Y_2' \lambda - (1 \leftrightarrow 2). \quad (5.1)$$

Charges become integrable and conserved: the second, non integrable part vanishes while in the first line of (4.5),  $E, A_\phi^0$  get replaced by  $\bar{E}, \bar{A}_\phi^0$ . In order to see this,



one has to go back to (4.1) where the second term now contributes to remove the  $u$ -dependent terms when using that, on shell,  $E = \bar{E}(\phi) - u\lambda(\phi)Y'$ , and  $A_\phi^0 = \bar{A}_\phi^0(\phi) - u\lambda'$ . Finally, the field dependent central charge becomes

$$K_{s_1, s_2} = \frac{1}{8\pi G} \int_0^{2\pi} d\phi \left[ Y_1' T_2'' + \lambda^2 T_1 Y_2' - (1 \leftrightarrow 2) \right]. \quad (5.2)$$

## 6 Discussion

Apart from its intrinsic interest, one might hope that the elaborate symmetry structure and the explicit solution of the three-dimensional Einstein-Maxwell system with non trivial asymptotics at Scri presented here could be suitably tuned so as to have applications in the context of holographic condensed matter models in 1+1 dimensions. Indeed, the Einstein-Maxwell system with various backgrounds, asymptotics, and additional scalar or form fields is ubiquitous in this context, see for instance [32–34], and more specifically [35–37] in three bulk dimensions. From the viewpoint of symmetries as well, this is quite reasonable since the  $\mathfrak{bms}_3$  algebra is isomorphic to  $\mathfrak{gca}_2$ , the Galilean conformal algebra in 2 dimensions [38, 39].

Independently of such speculations, let us compare the three-dimensional results derived here to those of the four dimensional case. First, we note that in the four dimensional Einstein-Maxwell system, one imposes the conditions  $A_r = 0$  and  $A_u = O(r^{-1})$  (see e.g. [4] and [40] section II.C for a detailed discussion of pure electromagnetism). As shown in [25], from the viewpoint of asymptotic symmetries, the absence of a term in  $A_u$  of order zero in  $r^{-1}$  also guarantees a time independent symmetry algebra similar to the one discussed here in three dimensions, but with an additional arbitrary dependence on the supplementary polar angle. In four dimensions, the electromagnetic news nevertheless persists since it is encoded in different components of the vector potential.

Concerning the algebra of charges, there does not exist, to our knowledge, a complete study of the Einstein-Maxwell system in the four-dimensional case. That is the reason why we compare the rest of the results here to the purely gravitational ones in four dimensions.

As recalled in the introduction, in four dimensions, self-consistent asymptotically flat solution spaces at Scri including charged black hole solutions have been constructed in spaces involving integer powers of  $1/r$ .

The simplest solution in three dimensions is the flat limit of the charged BTZ black hole. It is characterized by  $\omega = 0 = A_\phi^0 = A_{mn} = A_u^0$  and  $\theta = 8GM$ ,  $\chi = 8GJ$ ,  $\lambda = 2GQ$ , where  $M, J, Q$  are constants that, according to (4.5), are interpreted as the mass, angular momentum and electric charge of the solution. Let us recall that

the uncharged solutions with  $Q = 0$  describe three dimensional cosmologies [20–22] when  $M \geq 0$  and spinning particles, i.e., the angular defects and excesses of [18], when  $M < 0$ .

In both cases, it is the news, electromagnetic in the former and gravitational in the latter, that is responsible for the non-integrability and non conservation of the charges. In the latter case, there is no (field-dependent) central extension, unless one admits singular symmetry generators at null infinity and considers a local extension of the  $\mathfrak{bms}_4$  algebra including superrotations [11–13]. It disappears when switching off the news, as this reduces super to standard Lorentz rotations. In the former case, superrotations always exist and are globally well-defined at null infinity. The field dependent central extension persists even after switching off the electromagnetic news. To our knowledge, this is the first example in the context of asymptotic symmetries where there is a field dependent term in the symmetry algebra and in the central extension of the algebra of conserved and integrable charges.

The first field independent term in (5.2) exists also for pure gravity in three dimensions and is well understood from a cohomological point of view [16, 41]. It has also been used in an argument pertaining to the Bekenstein-Hawking entropy of the three-dimensional cosmological solutions [42, 43], modeled on the one in [44] for the BTZ black holes.

The second field dependent term involving the electric charge aspect, is novel and much less understood. It certainly deserves further study, both from the viewpoint of Lie algebroid cohomology and from a physical perspective.

Finally, let us comment on the nature of the solutions considered here. As in the majority of the papers on the subject since the pioneering work by Bondi et al. [1], the solutions are constructed as formal power series in the radial coordinate. In the polyhomogeneous case, there has been an investigation of convergence and existence of such solutions for linear massless higher spin fields on Minkowski spacetime as a preliminary study for the gravitational problem [45]. Addressing this question is clearly relevant in this set-up as well, but beyond the scope of the current work. We just note that the asymptotic symmetry algebra itself is not very sensitive to the details of solution space, as it is based solely on (2.1), (2.2) and the absence of news in later considerations.

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## A Details on computations

### A.1 Residual symmetries

Gauge parameters  $\xi^\mu$  that preserve the metric ansatz depend on two arbitrary functions  $T(\phi), Y(\phi)$ :

- $\mathcal{L}_\xi g_{rr} = 0$  implies  $\partial_r \xi^u = 0$  and so  $\xi^u = f(u, \phi)$ ,
- $\mathcal{L}_\xi g_{r\phi} = 0$  implies  $\partial_r \xi^\phi = \frac{e^{2\beta}}{r^2} \partial_\phi f$  and so  $\xi^\phi = Y(u, \phi) - \int_r^\infty dr' \frac{e^{2\beta}}{r'^2} \partial_\phi f$ ,
- $\mathcal{L}_\xi g_{\phi\phi} = 0$  implies  $\xi^r = -r(\partial_\phi \xi^\phi - U \partial_\phi f)$ ,
- $\mathcal{L}_\xi g_{u\phi} = o(r)$  implies  $\partial_u Y = 0$  and so  $Y = Y(\phi)$ ,
- $\mathcal{L}_\xi g_{ur} = o(r^0)$  implies  $\partial_u f = \partial_\phi Y$  and so  $f = T(\phi) + u \partial_\phi Y$ ,
- $\mathcal{L}_\xi g_{uu} = o(r)$  implies no further conditions.

The gauge parameter  $\epsilon$  preserving the gauge and fall-off conditions of the gauge potentials depends on an arbitrary function  $E(u, \phi)$  according to

- $\mathcal{L}_\xi A_r + \partial_r \epsilon = 0$  implies  $\epsilon = E(u, \phi) + \partial_\phi \xi^u \int_r^\infty \frac{e^{2\beta} A_\phi}{r'^2} dr'$ ,
- $\mathcal{L}_\xi A_u + \partial_u \epsilon = O(\ln \frac{r}{r_0}) = \mathcal{L}_\xi A_\phi + \partial_\phi \epsilon$  imply no further conditions.

### A.2 Asymptotic symmetry algebra

We want to show that the gauge parameters (2.6), when equipped with the bracket (2.5), provide a representation of the Lie algebra (2.7). By evaluating  $\mathcal{L}_\xi g_{\mu\nu}$ , we find

$$\begin{cases} -\delta\beta = \xi^\alpha \partial_\alpha \beta + \frac{1}{2}[\partial_u f + \partial_r \xi^r + \partial_\phi f U], \\ -\delta U = \xi^\alpha \partial_\alpha U + U[\partial_u f + \partial_\phi f U - \partial_\phi \xi^\phi] - \partial_u \xi^\phi - \partial_r \xi^\phi V + \partial_\phi \xi^r \frac{e^{2\beta}}{r^2}, \end{cases} \quad (\text{A.1})$$

while  $-\delta A_\phi = \xi^\alpha \partial_\alpha A_\phi + A_\alpha \partial_\phi \xi^\alpha + \partial_\phi \epsilon$ . It follows that

$$\begin{cases} \delta_1 \xi_2^u = 0, \\ \delta_1 (\partial_r \xi_2^\phi) = \partial_\phi f_2 \frac{e^{2\beta}}{r^2} 2\delta_1 \beta, \\ \delta_1 \xi_2^r = -r [\partial_\phi (\delta_1 \xi_2^\phi) - \partial_\phi f_2 \delta_1 U], \\ \delta_1 (\partial_r \epsilon_2) = -\frac{1}{r^2} (\partial_\phi f_2 e^{2\beta} \delta_1 A_\phi + \partial_\phi f_2 e^{2\beta} A_\phi 2\delta_1 \beta). \end{cases} \quad (\text{A.2})$$

Direct computation then shows that

$$\begin{aligned} \partial_r \hat{\xi}^u &= \partial_r \hat{f} = 0, & \partial_u \hat{f} &= \partial_\phi \hat{Y}, & \hat{f} &= \hat{T} + u \partial_\phi \hat{Y}, \\ \partial_r \hat{\xi}^\phi &= \frac{e^{2\beta} \hat{f}}{r^2}, & \lim_{r \rightarrow \infty} \hat{\xi}^\phi &= \hat{Y}, \\ \hat{\xi}^r &= -\partial_\phi \hat{\xi}^\phi + U \partial_\phi \hat{f}, \\ \partial_r \hat{\epsilon} &= -\frac{\partial_\phi \hat{f} e^{2\beta} A_\phi}{r^2}, & \lim_{r \rightarrow \infty} \hat{\epsilon} &= \hat{E}, \end{aligned}$$

which proves the result since these conditions determine uniquely gauge parameters (2.6) where  $(T, Y, E)$  have been replaced by  $(\hat{T}, \hat{Y}, \hat{E})$ .

### A.3 Solution space

The equations of motion can be organized as follows

$$\partial_\nu (\sqrt{-g} F^{\mu\nu}) = 0, \quad (\text{A.3})$$

$$\partial_\nu (\sqrt{-g} F^{\phi\nu}) = 0, \quad (\text{A.4})$$

$$\partial_\nu (\sqrt{-g} F^{r\nu}) = 0, \quad (\text{A.5})$$

$$L_{r\alpha} = G_{r\alpha} - T_{r\alpha} = 0, \quad (\text{A.6})$$

$$L_{\phi\phi} = G_{\phi\phi} - T_{\phi\phi} = 0, \quad (\text{A.7})$$

$$L_{u\phi} = G_{u\phi} - T_{u\phi} = 0, \quad (\text{A.8})$$

$$L_{uu} = G_{uu} - T_{uu} = 0. \quad (\text{A.9})$$

When equations (A.3) and (A.4) hold, the electromagnetic Bianchi equation reduces to  $\partial_r [\partial_\nu (\sqrt{-g} F^{r\nu})] = 0$ . This means that if  $\partial_\nu (\sqrt{-g} F^{r\nu}) = 0$  for some constant  $r$ , it vanishes for all  $r$ . The gravitational Bianchi identities can be written as

$$0 = 2\sqrt{-g} \nabla_\nu G_\mu^\nu = 2\partial_\nu (\sqrt{-g} L_\mu^\nu) + \sqrt{-g} L_{\rho\sigma} \partial_\mu g^{\rho\sigma} + 2\sqrt{-g} \nabla_\nu T_\mu^\nu. \quad (\text{A.10})$$

When (A.3)-(A.6) are satisfied and  $\mu = r$  in (A.10), one gets  $L_{\phi\phi} \partial_r g^{\phi\phi} = 0$  which implies  $L_{\phi\phi} = 0$ . In this case, the remaining Bianchi identities reduce to  $2\partial_\nu (\sqrt{-g} L_\phi^\nu) = 0 = 2\partial_\nu (\sqrt{-g} L_u^\nu)$ . The first one gives  $\partial_r (r L_{u\phi}) = 0$ . This means that if  $r L_{u\phi} = 0$  for some fixed  $r$ , it vanishes everywhere. Finally, when  $L_{u\phi} = 0$ , the last Bianchi identity reads  $\partial_r (r L_{uu}) = 0$ . Thus the only non-vanishing term of  $r L_{uu}$  is the constant one.

Accordingly, the equations of motions are solved in the following order:

- 4 main equations:  $L_{rr} = 0, \partial_\nu(\sqrt{-g}F^{u\nu}) = 0, L_{r\phi} = 0, L_{ru} = 0,$
- 1 standard equation:  $\partial_\nu(\sqrt{-g}F^{\phi\nu}) = 0,$
- 3 supplementary equations:  $\partial_\nu(\sqrt{-g}F^{r\nu}) = 0, L_{u\phi} = 0, L_{uu} = 0,$
- 1 trivial equation:  $L_{\phi\phi} = 0.$

Starting with  $L_{rr} = 0$ , we have  $g_{rr} = 0$ ,  $R_{rr} = 2\frac{\partial_r\beta}{r}$ ,  $T_{rr} = \frac{2}{r^2}(F_{r\phi})^2$ . Hence  $\partial_r\beta = \frac{1}{r}(F_{r\phi})^2$  and thus  $\beta = \beta_0(u, \phi) - \int_r^\infty dr' \frac{1}{r'}(F_{r\phi})^2$  with  $\beta_0$  an integration constant. The fall-off condition  $\beta = o(r^0)$  puts  $\beta_0$  to zero and thus,

$$\beta = - \int_r^\infty dr \frac{1}{r'}(F_{r\phi})^2. \quad (\text{A.11})$$

Consider now the equation  $\partial_\nu(\sqrt{-g}F^{u\nu}) = 0$ . Explicitly, this equation reads  $\partial_r(re^{2\beta}F^{ur}) + \partial_\phi(re^{2\beta}F^{u\phi}) = 0$ . Defining

$$m := e^{2\beta}F^{ur} = -e^{-2\beta}(F_{ur} - UF_{r\phi}), \quad (\text{A.12})$$

and using  $e^{2\beta}F^{u\phi} = -\frac{1}{r^2}F_{r\phi}$ , this equation of motion is a first order differential equation for  $m$ ,

$$\partial_r(rm) = \frac{\partial_\phi F_{r\phi}}{r} \implies m = \frac{-\lambda - \int_r^\infty dr' \frac{\partial_\phi F_{r\phi}}{r'}}{r}, \quad (\text{A.13})$$

with  $\lambda(u, \phi)$  a constant of integration.

For  $L_{r\phi} = 0$ , we have  $g_{r\phi} = 0$ ,  $R_{r\phi} = -\partial_{\phi r}\beta + \frac{\partial_\phi\beta}{r} - r^2e^{-2\beta}\partial_r\beta\partial_rU + \frac{3}{2}re^{-2\beta}\partial_rU + \frac{r^2}{2}e^{-2\beta}\partial_{rr}U$ ,  $T_{r\phi} = 2F_{r\phi}m$ . Defining

$$n := \frac{r^2}{2}e^{-2\beta}\partial_rU, \quad (\text{A.14})$$

$R_{r\phi} = -\partial_{\phi r}\beta + \frac{\partial_\phi\beta}{r} + \left(\partial_r + \frac{1}{r}\right)n$ , the equation is a first order differential equation for  $n$ ,

$$\begin{aligned} \partial_r n + \frac{n}{r} &= 2F_{r\phi}m + \partial_{r\phi}\beta - \frac{\partial_\phi\beta}{r} \\ \implies n &= \frac{N - 2 \int_r^\infty dr' r'(2F_{r\phi}m + \partial_{r\phi}\beta - \frac{\partial_\phi\beta}{r})}{2r}, \end{aligned} \quad (\text{A.15})$$

with  $N(u, \phi)$  an integration constant. As a consequence of the fall-off condition on  $U$ , we end up with

$$U = - \int_r^\infty dr' \left( \frac{2e^{2\beta}}{r'^2} n \right). \quad (\text{A.16})$$

For  $L_{ur} = 0$ , we have  $G_{ru} = -\frac{1}{2}g_{ru}(R_{\phi\phi}g^{\phi\phi} + 2R_{r\phi}g^{r\phi} + R_{rr}g^{rr})$ ,  $R_{\phi\phi} = re^{-2\beta}(\partial_r V + 2\partial_\phi U) - 2\partial_\phi\phi\beta + r^2e^{-2\beta}\partial_\phi r U - 2(\partial_\phi\beta)^2 - \frac{e^{-4\beta}}{2}r^4(\partial_r U)^2$  and  $-2r^2(T_{ru}g^{ru} + T_{r\phi}g^{r\phi} + \frac{1}{2}T_{rr}g^{rr}) = 2r^2m^2$ . This gives

$$\partial_r V = 2re^{2\beta}m^2 + \frac{2e^{2\beta}\partial_\phi\phi\beta}{r} - r\partial_\phi r U + \frac{2e^{2\beta}(\partial_\phi\beta)^2}{r} + \frac{1}{2}e^{-2\beta}r^3(\partial_r U)^2 - 2\partial_\phi U, \quad (\text{A.17})$$

and

$$V = \theta - \int_r^\infty dr' \left( 2re^{2\beta}m^2 + \frac{2e^{2\beta}\partial_\phi\phi\beta}{r} - r\partial_\phi r U + \frac{2e^{2\beta}(\partial_\phi\beta)^2}{r} + \frac{1}{2}e^{-2\beta}r^3(\partial_r U)^2 - 2\partial_\phi U \right), \quad (\text{A.18})$$

with  $\theta(u, \phi)$  a constant of integration.

For  $j \geq 0$ , we have

$$\partial_r[r^i \ln^j r] = \begin{cases} \sum_{k=0}^j C_{ijk} r^{i-1} \ln^k r, & i \neq 0, \\ jr^{-1} \ln^{j-1} r, & i = 0, \end{cases} \quad (\text{A.19})$$

$$\int r^i \ln^j r \, dr = \begin{cases} \sum_{k=0}^j D_{ijk} r^{i+1} \ln^k r, & i \neq -1, \\ \frac{\ln^{j+1} r}{j+1}, & i = -1, \end{cases} \quad (\text{A.20})$$

for some coefficients  $C_{ijk}$  and  $D_{ijk}$ , and up to constants for the integrations. Consider then series  $\mathcal{S}^n$  with elements of the form

$$\sum_{i \leq -n, 0 \leq j \leq -i-n} s_{ij}(u, \phi) r^i \ln^j r, \quad (\text{A.21})$$

with  $n \geq 0$ . These series satisfy  $\mathcal{S}^{n+1} \subset \mathcal{S}^n$ ,  $\mathcal{S}^n * \mathcal{S}^m \subset \mathcal{S}^{n+m}$ ,  $(\mathcal{S}^n)' \subset \mathcal{S}^{n+1}$ . For integration however,  $\int dr \mathcal{S}^{n+1} \subset \mathcal{S}^n$ , up to constants for  $n \neq 0$  and the divergent logarithmic term for  $n = 0$ .

The ansatz (3.3) belongs to  $\mathcal{S}^0$ , up to the divergent logarithmic term proportional to  $\alpha(u, \phi)$ . This implies  $F_{r\phi} \in \mathcal{S}^1$  and  $\beta$  in (A.11), because of the absence of the constant, belongs to  $\mathcal{S}^2$  with all coefficients determined by the coefficients  $\alpha(u, \phi)$ ,  $A_{mn}(u, \phi)$  of (3.3).

In the same way, from (A.13), it follows that  $m \in \mathcal{S}^1$  with all coefficients determined by those of (3.3) and the integration function  $\lambda$ .

For  $U$ , we have in a first stage that  $n$  belongs to  $\mathcal{S}^0$  and is determined by the data in (3.3) and the integration constants  $\lambda, N$ . For  $U$  itself, it follows from (A.16), that it belongs to  $\mathcal{S}^1$ , with no new integration constant because of the assumed fall-off.

Finally, it follows from (A.18) that  $V$  belongs to  $\mathcal{S}^0$ , up to a logarithmic divergence, with coefficients determined by the data in (3.3) and the integration functions  $\alpha, \lambda, N, \theta$ .

In summary, by integrating  $m$  in  $r$  in order to get  $A_u$  and making the  $\alpha$  dependence explicit, we find that all main equations are solved as

$$m = -\frac{\lambda}{r} - \frac{\alpha'}{r^2} + \sum_{m=1}^{\infty} \sum_{n=0}^m \frac{m_{mn} (\ln \frac{r}{r_0})^n}{r^{m+2}}, \quad (\text{A.22})$$

$$\beta = -\frac{\alpha^2}{2r^2} + \sum_{m=1}^{\infty} \sum_{n=0}^m \frac{\beta_{mn} (\ln \frac{r}{r_0})^n}{r^{m+2}}, \quad (\text{A.23})$$

$$U = \frac{4\lambda\alpha \ln \frac{r}{r_0} + 2\lambda\alpha - N}{2r^2} + \sum_{m=1}^{\infty} \sum_{n=0}^m \frac{U_{mn} (\ln \frac{r}{r_0})^n}{r^{m+2}}, \quad (\text{A.24})$$

$$A_u = -\lambda \ln \frac{r}{r_0} + A_u^0 + \frac{\alpha'}{r} + \sum_{m=1}^{\infty} \sum_{n=0}^m \frac{B_{mn} (\ln \frac{r}{r_0})^n}{r^{m+1}}, \quad (\text{A.25})$$

$$V = 2\lambda^2 \ln \frac{r}{r_0} + \theta + \frac{2\alpha\lambda' - 2\lambda\alpha'}{r}, \quad (\text{A.26})$$

$$+ \sum_{m=1}^{\infty} \sum_{n=0}^m \frac{V_{mn} (\ln \frac{r}{r_0})^n}{r^{m+1}}, \quad (\text{A.27})$$

where  $m_{mn}, \beta_{mn}, U_{mn}, V_{mn}, B_{mn}$  are determined by  $\alpha(u, \phi)$ ,  $A_{mn}(u, \phi)$ , the integration constants  $\lambda(u, \phi), N(u, \phi)$  and their  $\phi$  derivatives.

The standard equation determines the  $u$  evolution of  $\alpha, A_\phi^0$  and  $A_{mn}(u, \phi)$ . Indeed,  $\partial_\nu(\sqrt{-g}F^{\phi\nu}) = \partial_u(re^{2\beta}F^{\phi u}) + \partial_r(re^{2\beta}F^{\phi r}) = 0$ . Since  $e^{2\beta}F^{\phi r} = Um + \frac{1}{r^2}(F_{u\phi} + VF_{r\phi})$ ,  $e^{2\beta}F^{\phi u} = \frac{1}{r^2}F_{r\phi}$ , we get

$$\partial_u F_{r\phi} = -r^2 \left( \partial_r + \frac{1}{r} \right) \left[ Um + \frac{1}{r^2} (F_{u\phi} + VF_{r\phi}) \right],$$

which is a differential equation governing the  $u$ -dependence of  $F_{r\phi}$  and thus of  $A_\phi$ . In terms of coefficients, we get

$$\dot{\alpha} = -\lambda', \quad \dot{A}_\phi^0 = -\lambda' + (A_u^0)', \quad \dot{A}_{mn+1} = \frac{(2m+1)\dot{A}_{mn} + X_{mn}}{2(n+1)}, \quad (\text{A.28})$$

where  $A_{mn+1} = 0$  when  $n = m$  and  $X_{mn}$  is a linear combination of  $\alpha, A_\phi^0, A_{mn}$ , integration functions  $\lambda, A_u^0, N, \theta$  and their  $\phi$  derivative.

The first supplementary equation reads explicitly  $0 = \partial_\nu(\sqrt{-g}F^{r\nu}) = \partial_u(re^{2\beta}F^{ru}) + \partial_\phi(re^{2\beta}F^{r\phi})$ . Since  $e^{2\beta}F^{ru} = -m = \frac{\lambda}{r} + O(r^{-2})$  and  $e^{2\beta}F^{r\phi} = \left[Um - \frac{1}{r^2}\left(F_{u\phi} + \frac{V}{r}F_{r\phi}\right)\right] = O(r^{-2})$ ,  $\lim_{r \rightarrow \infty} \partial_\nu(\sqrt{-g}F^{r\nu}) = 0$  implies  $\dot{\lambda} = 0$  so that  $\lambda = \lambda(\phi)$ . The first of equation (A.28) then implies  $\alpha = -\omega(\phi) - u\lambda'$ .

For the second supplementary equation,  $L_{u\phi} = 0$ , we have  $L_{u\phi} = \frac{1}{2r}(\theta' - \dot{N}) + O(r^{-2})$ . Hence,  $\lim_{r \rightarrow \infty}(rL_{u\phi}) = 0$  implies  $\dot{N} = \theta'$ .

For the last supplementary equation  $L_{uu} = 0$ , we have  $L_{uu} = \frac{\dot{\theta}}{r} + O(r^{-2})$ .  $\lim_{r \rightarrow \infty}(rL_{uu}) = 0$  then implies  $\partial_u\theta = 0$  and thus  $\theta = \theta(\phi)$  and then also that  $N = \chi(\phi) + u\theta'$ .

## References

- [1] H. Bondi, M. G. van der Burg, and A. W. Metzner, “Gravitational waves In general relativity. 7. Waves from axi-symmetric isolated systems,” *Proc. Roy. Soc. Lond. A* **269** (1962) 21.
- [2] R. K. Sachs, “Gravitational waves in general relativity. 8. Waves in asymptotically flat space-time,” *Proc. Roy. Soc. Lond. A* **270** (1962) 103.
- [3] E. Newman, L. Tamubrino, and T. Unti, “Empty space generalization of the Schwarzschild metric,” *J. Math. Phys.* **4** (1963) 915.
- [4] M. G. J. van der Burg, “Gravitational Waves in General Relativity. 10. Asymptotic Expansions for the Einstein-Maxwell Field,” *Proc. Roy. Soc. Lond. A* **310** (1969) 221–230.
- [5] A. Exton, E. Newman, and R. Penrose, “Conserved quantities in the Einstein-Maxwell theory,” *J. Math. Phys.* **10** (1969) 1566–1570.
- [6] J. Winicour, “Logarithmic asymptotic flatness,” *Foundations of Physics* **15** no. 5, (05, 1985) 605–616. <http://dx.doi.org/10.1007/BF01882485>.
- [7] P. T. Chrusciel, M. A. H. MacCallum, and D. B. Singleton, “Gravitational waves in general relativity. 14. Bondi expansions and the polyhomogeneity of Scri,” *Proc. Roy. Soc. Lond. A* **436** (1992) 299–316, [arXiv:gr-qc/9305021](https://arxiv.org/abs/gr-qc/9305021).
- [8] J. A. Valiente Kroon, “Logarithmic Newman-Penrose constants for arbitrary polyhomogeneous space-times,” *Class. Quant. Grav.* **16** (1999) 1653–1665, [arXiv:gr-qc/9812004](https://arxiv.org/abs/gr-qc/9812004) [gr-qc].



- [9] R. M. Wald and A. Zoupas, “A general definition of conserved quantities in general relativity and other theories of gravity,” *Phys. Rev.* **D61** (2000) 084027, [gr-qc/9911095](#).
- [10] G. Barnich and C. Troessaert, “Symmetries of asymptotically flat 4 dimensional spacetimes at null infinity revisited,” *Phys.Rev.Lett.* **105** (2010) 111103, [arXiv:0909.2617 \[gr-qc\]](#).
- [11] G. Barnich and C. Troessaert, “Aspects of the BMS/CFT correspondence,” *JHEP* **05** (2010) 062, [arXiv:1001.1541 \[hep-th\]](#).
- [12] G. Barnich and C. Troessaert, “BMS charge algebra,” *JHEP* **1112** (2011) 105, [arXiv:1106.0213 \[hep-th\]](#).
- [13] G. Barnich and C. Troessaert, “Comments on holographic current algebras and asymptotically flat four dimensional spacetimes at null infinity,” *JHEP* **1311** (2013) 003, [arXiv:1309.0794 \[hep-th\]](#).
- [14] A. Ashtekar, J. Bicak, and B. G. Schmidt, “Asymptotic structure of symmetry-reduced general relativity,” *Phys. Rev. D* **55** no. 2, (Jan, 1997) 669–686.
- [15] G. Compère, L. Donnay, P.-H. Lambert, and W. Schulgin, “Liouville theory beyond the cosmological horizon,” *JHEP* **03** (2015) 158, [arXiv:1411.7873 \[hep-th\]](#).
- [16] G. Barnich and G. Compère, “Classical central extension for asymptotic symmetries at null infinity in three spacetime dimensions,” *Class. Quant. Grav.* **24** (2007) F15, [gr-qc/0610130](#). Corrigendum: *ibid* 24 (2007) 3139.
- [17] J. D. Brown and M. Henneaux, “Central charges in the canonical realization of asymptotic symmetries: An example from three-dimensional gravity,” *Commun. Math. Phys.* **104** (1986) 207.
- [18] S. Deser, R. Jackiw, and G. ’t Hooft, “Three-dimensional Einstein gravity: Dynamics of flat space,” *Ann. Phys.* **152** (1984) 220.
- [19] K. Ezawa, “Transition amplitude in (2+1)-dimensional Chern-Simons gravity on a torus,” *Int.J.Mod.Phys.* **A9** (1994) 4727–4746, [arXiv:hep-th/9305170 \[hep-th\]](#).
- [20] L. Cornalba and M. S. Costa, “A New cosmological scenario in string theory,” *Phys.Rev.* **D66** (2002) 066001, [arXiv:hep-th/0203031 \[hep-th\]](#).

- [21] L. Cornalba and M. S. Costa, “Time dependent orbifolds and string cosmology,” *Fortsch.Phys.* **52** (2004) 145–199, [arXiv:hep-th/0310099](#) [hep-th].
- [22] G. Barnich, A. Gomberoff, and H. A. Gonzalez, “Flat limit of three dimensional asymptotically anti-de Sitter spacetimes,” *Phys.Rev.* **D86** (2012) 024020, [arXiv:1204.3288](#) [gr-qc].
- [23] C. Martinez, C. Teitelboim, and J. Zanelli, “Charged rotating black hole in three space-time dimensions,” *Phys.Rev.* **D61** (2000) 104013, [arXiv:hep-th/9912259](#) [hep-th].
- [24] R. K. Sachs, “Asymptotic symmetries in gravitational theory,” *Phys. Rev.* **128** (1962) 2851–2864.
- [25] G. Barnich and P.-H. Lambert, “Einstein-Yang-Mills theory: Asymptotic symmetries,” *Phys.Rev.* **D88** (2013) 103006, [arXiv:1310.2698](#) [hep-th].
- [26] L. A. Tamburino and J. H. Winicour, “Gravitational fields in finite and conformal Bondi frames,” *Phys. Rev.* **150** (1966) 1039.
- [27] I. M. Anderson and C. G. Torre, “Asymptotic conservation laws in field theory,” *Phys. Rev. Lett.* **77** (1996) 4109–4113, [hep-th/9608008](#).
- [28] G. Barnich and F. Brandt, “Covariant theory of asymptotic symmetries, conservation laws and central charges,” *Nucl. Phys.* **B633** (2002) 3–82, [hep-th/0111246](#).
- [29] G. Barnich, “Boundary charges in gauge theories: Using Stokes theorem in the bulk,” *Class. Quant. Grav.* **20** (2003) 3685–3698, [hep-th/0301039](#).
- [30] G. Barnich and G. Compère, “Surface charge algebra in gauge theories and thermodynamic integrability,” *J. Math. Phys.* **49** (2008) 042901, [arXiv:0708.2378](#) [gr-qc].
- [31] G. Barnich and G. Compère, “Conserved charges and thermodynamics of the spinning Goedel black hole,” *Phys. Rev. Lett.* **95** (2005) 031302, [hep-th/0501102](#).
- [32] S. A. Hartnoll, C. P. Herzog, and G. T. Horowitz, “Building a Holographic Superconductor,” *Phys. Rev. Lett.* **101** (2008) 031601, [arXiv:0803.3295](#) [hep-th].
- [33] S. Kachru, X. Liu, and M. Mulligan, “Gravity duals of Lifshitz-like fixed points,” *Phys. Rev.* **D78** (2008) 106005, [arXiv:0808.1725](#) [hep-th].

- [34] K. Balasubramanian and J. McGreevy, “Gravity duals for non-relativistic CFTs,” *Phys. Rev. Lett.* **101** (2008) 061601, [arXiv:0804.4053 \[hep-th\]](#).
- [35] D. Maity, S. Sarkar, N. Sircar, B. Sathiapalan, and R. Shankar, “Properties of CFTs dual to Charged BTZ black-hole,” *Nucl. Phys.* **B839** (2010) 526–551, [arXiv:0909.4051 \[hep-th\]](#).
- [36] J. Ren, “One-dimensional holographic superconductor from AdS<sub>3</sub>/CFT<sub>2</sub> correspondence,” *JHEP* **11** (2010) 055, [arXiv:1008.3904 \[hep-th\]](#).
- [37] K. Jensen, “Chiral anomalies and AdS/CMT in two dimensions,” *JHEP* **01** (2011) 109, [arXiv:1012.4831 \[hep-th\]](#).
- [38] A. Bagchi, R. Gopakumar, I. Mandal, and A. Miwa, “GCA in 2d,” *JHEP* **08** (2010) 004, [arXiv:0912.1090 \[hep-th\]](#).
- [39] A. Bagchi, “Correspondence between Asymptotically Flat Spacetimes and Nonrelativistic Conformal Field Theories,” *Phys.Rev.Lett.* **105** (2010) 171601.
- [40] A. Ashtekar, “Asymptotic Quantization: Based on 1984 Naples Lectures,” Naples, Italy: Bibliopolis (1987) 107 p. (Monographs and textbooks in physical science, 2).
- [41] G. Barnich and C. Troessaert, “Supertranslations call for superrotations,” *PoS CNCFG2010* (2010) 010, [arXiv:1102.4632 \[gr-qc\]](#).
- [42] G. Barnich, “Entropy of three-dimensional asymptotically flat cosmological solutions,” *JHEP* **1210** (2012) 095, [arXiv:1208.4371 \[hep-th\]](#).
- [43] A. Bagchi, S. Detournay, R. Fareghbal, and J. Simon, “Holography of 3d Flat Cosmological Horizons,” *Phys.Rev.Lett.* **110** (2013) 141302, [arXiv:1208.4372 \[hep-th\]](#).
- [44] A. Strominger, “Black hole entropy from near-horizon microstates,” *JHEP* **02** (1998) 009, [arXiv:hep-th/9712251](#).
- [45] J. A. Valiente-Kroon, “On the existence and convergence of polyhomogeneous expansions of zero rest mass fields,” *Class.Quant.Grav.* **17** (2000) 4365–4376, [arXiv:gr-qc/0005087 \[gr-qc\]](#).